MATHEMATICAL MODELING OF NONSTATIONARY SEPARATION FLOW
AROUND A GRID OF HEAT EXCHANGER TUBES
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To estimate the intensification of the surface heat- and mass-transfer process in tubular heat-exchange apparatus, a mathematical model is proposed for the analysis of the frequency and scalar characteristics of the force action of the flow of a single-phase heat carrier in the tube grid.

The hydrodynamic vibration of heat-exchanger tubes can become the reason for the premature failure of heat-exchanger apparatus, and can contract the range of allowable modes of operation of the installation. To prevent these vibrations it is necessary to be able to determine the magnitude and frequency spectrum of the hydrodynamic forces acting on the tubes for different excitation mechanisms.

In a number of cases a sufficiently efficient model for the analysis of the collapsing flow around bodies at high Reynolds numbers turns out to be the model of a perfect medium with the approximation of a nonstationary vortex layer on the body and a system of discrete point vortices behind it [1]. For poorly streamlined bodies that have no sharp edges (a cylinder, for example), the flow separation points on the body surface are not known and depend on the number Re in a viscous fluid. This requires development of supplements to the available models. The problem of the separation flow around a single cylinder has been investigated most and it is examined in $[2-4]$ on an ideal fiuid model.

Problems of viscous fluid flow around obstacles are also studied by using finite-difference schemes [5, 6]. However, these methods are suitable for small Reynolids numbers and require a significantly high machine time expenditure.

The model proposed in [3] to analyze the collapsing flow around a Eube takes into account the influence of the Reynolds number by selecting the position of the boundary-layer separation point from the tube surface. The model is also applied to solve the following problems: separation flow around a tube fluctuating both along and across the stream, tubes in a selfoscillation mode. The computed quantitative characteristics obtained in all cases (the values of the time-varying coefficients of frontal drag and lift, the width of the pulling zone, the amplitude-frequency curves) are in agreement with the numerous experimental data in the $2 \cdot 10^{3}-$ $5 \cdot 10^{6}$ Reynolds number range.

An approach based on a combination of the model for an ideal medium and boundary-layer theory is used in [4] to compute the separation flow around a cylinder. Empirical data which are ordinarily used in standard boundary-layer models are relied upon here.

Below, the problem of the separation flow around a number of tubes that has certain differences in principle as compared with the problem of a single tube is considered by using the model in [3]. The results of a computation can have numerous practical applications (see [7], say).

Let an infinite series of fixed tubes be flowed around with separation by a plane fluid flow moving normally to the front of the grid. At certain points in a viscous fluid, the boundary layer separates from the cylinder surface and forms turbulent vortex layers that rotate in large scale Karman street eddies.

Let us examine the schematization of this phenomenon by model for the plane flow of an ideal incompressible fluid. We assume that the vortex sheet sheds at points of separation A and consists of fluid particles incident from the boundary layer onto the frontal part of
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the cylinder (Fig. 1). A counter motion exists on the root part of the cylinder, and consequently, in the model we assume that vorticity of the opposite sign mixes with the main vortex sheet (presumably in the triangle $A B C$ ). This model for a single tube is given a foundation in more detail in [3].

We now note certain fundamental features that distinguish this problem from the problem of separation flow around a single tube.

The flow field is periodic although the period is certainly not the grid spacing t since vortex separation on adjacent tubes can occur with a phase shift. In the case of fixed tubes the phase shifts $\beta=0, \beta=\pi$ are of greatest practical interest. The second essential singularity is that the free vortices being separated form an infinite chain whose axes are parallel to the grid front. No matter how far this chain is removed from its site of origination, the velocities it induces in the whole flow plane do not tend to zero. This problem is more complex than the problem of the flow around a single tube since the application of the mirror image method would result in an infinite reflection process. Direct application of the method of attached vortices located on the contour (Zhukovskii vortices) is difficult because of the nearby location of the trailing sheet to the cylinder surface. Consequently, a certain combined method will be proposed below.

The location of separation points on a system of tubes around which a viscous fluid flows depends on the numbers Re exactly as for a single tube. However, the singularity of this case is that separation occurs for $\alpha=90^{\circ}$ for the compact grids that are used in engineering. This is explained by the fact that a large negative pressure gradient is observed on the frontal part of the tube while a large positive pressure gradient that the boundary layer cannot overcome would occur at the root part in the absence of separation. Consequently, the coordinates of the separation points are taken equal to $\alpha= \pm 90^{\circ}$ for the subsequent computations, although computations can be executed by the same method even for another position of the points of separation as has been done for a single tube. It is meaningful to do this for sparse grids.

Computations are performed below for the phase shift $\beta=0$. The other case of practical interest $\beta=\pi$ can be computed by the same method; however, this will result in substantial growth of the volume of calculations.

Let us consider one chain of free vortices located behind a cylinder (Fig. 1). The velocity caused by such a vortex chain in the flow plane is

$$
\begin{equation*}
u-i v=\frac{\Delta \Gamma_{i}}{2 i t} \operatorname{cth} \frac{\pi\left(z-z_{i}\right)}{t}, z_{i}=x_{i}+i y_{i} \tag{1}
\end{equation*}
$$

where $z_{i}$ is the coordinate of a vortex in the $i-t h$ chain, and $\Delta \Gamma_{i}$ is the intensity of this vortex.

As $t \rightarrow \infty$ this dependence evidently expresses the velocity induced by a single vortex. If we let $x_{i}$ tend to infinity in (1), we then obtain

$$
\begin{equation*}
u=0 ; v=\frac{\Delta \Gamma_{i}}{2 t} \tag{2}
\end{equation*}
$$

Therefore, even if the free vortex chain were removed to infinity by the stream, it would still induce a finite vertical velocity. It should be noted that the mentioned limit values of the velocities (2) are achieved in practice as the vortex chain is removed the distance of one step from the cylinders. Since vortex chains having reverse circulations and approximately identical absolute magnitudes shed periodically from the cylinder during separation flow, then the far wake has practically no influence on the flow mode.

The boundary conditions assume the normal velocities on the tube surface equal zero. Because of the periodicity of the flow pattern, it is sufficient to satisfy the boundary conditions on one circumference. This can be achieved by different methods. In this case the most effective method seems to be the following combination. As in the case of a single tube, we place a dipole imaging the tube at the center of each circumference. Then the velocity caused by the main stream and the system of dipoles is determined by the formula

$$
\begin{equation*}
u-i v=u_{\infty}-A \frac{\left(\frac{\pi}{t}\right)^{2}}{\operatorname{sh}^{2} \frac{\pi z}{t}} \tag{3}
\end{equation*}
$$

As the step tends to infinity (3) will become the exact solution of the problem of the flow around a single tube. For a finite value of the step the expression obtained will be a good approximation.

Since the solution (3) is not exact, the stream function does not take on a constant value on the circumference. We determine the constant A by satisfying the boundary conditions at the points 1 and 2 (Fig. 1). Then the solution (3) will be exact for a grid of ovals close to circles. In particular, for $t=4$ and $r=4$ the ratio of the semiaxes of the ovals will be 1.017 .

In order to satisfy the boundary conditions on the tubes approximately in the presence of a free vortex chain, we take the mirror image of "its" free vortex in each circle (Fig. 1). The velocity caused by the free vortex chain and by the vortex chain obtained by the mentioned mirror imaging can be written by using (1)

$$
\begin{equation*}
u-i v=\frac{\Delta \Gamma_{i}}{2 i t} \operatorname{cth} \frac{\pi\left(z-z_{i}\right)}{t}-\frac{\Delta \Gamma_{i}}{2 i t} \cdot \operatorname{cth} \frac{\pi\left(z-z_{j i}\right)}{t} . \tag{4}
\end{equation*}
$$

Such a solution will again tend to the exact solution as $t \rightarrow \infty$ and is a good approximation for a finite value of the step if the free vortex chain is near the tube. In order to satisfy the boundary conditions more exactly, we apply the method of attached vortices. We arrange N free vortices on the tube and select their circulation in such a manner that the boundary conditions on the tube would be satisfied at the $\mathbb{N}$ given points. The intensities of the vortices located on the tube are obtained relatively small, as computations confirmed, since these vortices just correct a good first approximation.

Therefore, the stream flowing around the tube grid is comprised of the streams: a) planeparallel, moving with the velocity $u_{\infty} ; b$ ) caused by the dipole chain placed at the center of the tubes; c) caused by the system of free vortices located behind each tube; d) from "fictitious" vortices; e) caused by the attached Zhukovskii vortices located on the circle.

According to the Thomson theorem, the total circulation around a contour enclosing any of the tubes and its referred free vortices should equal zero. This could be achieved by placing a vortex with intensity equal to and opposite in sign to the sum of the vortex intensities on the tube within each tube. Instead of this, the boundary conditions on the tube could be satisfied at $N-1$ points, and the $N-t h$ condition could be to select the condition that the sum of the vortex circulations on the tube equals zero.

The most detailed experimental investigations of the regularities of vortex separation in tube bundles were conducted in [8]. Experimental data are presented in [8] for values of the dimensionless frequency of vortex separation $S h=f d / u$ in the bundle as a function of the relative step and the tube location in the beam. According to these data, the Strouhal number computed according to the mean fluid velocity between the tubes can be both greater and less than for the flow around a single tube. The experimental dependences $S h=\varphi(t / r)$ are presented in Fig. 2 for corridor bundles with $Z / r=6$ to $\infty$ (points 1 , curve 1 ) and for different $t / r$, and the same data for checkerboard bundles (points 2, curve 2).

The separation flow around one series of tubes was computed by the method described for the relative grid densities $t / r=3,4,5,6,10,100$. The corresponding points (points 3) are superposed in Fig. 2. The agreement between the computed data and curve 1 is explained by the fact that the second series in this experiment stood off by relative distance $2 / \mathrm{r}=$ 6 to $\infty$ from the first and, therefore, its influence was practically nonexistent.

Shown in Fig. 3 is the change in intensity of the vortex sheet being shed at the separation points $A$ (see Fig. 1) as a function of the dimensionless time $\bar{\tau}=\tau u_{\infty} / r$. Curve 1 corresponds to the relative grid compactness of $t / r=100$, and curve 2 to the compactness $t / r=$ 4. For $t / r=100$ the results of computing the flow around the tube grid agree with analogous results for a single tube, which is a check on the computation. It is seen from Fig. 3 that as the relative distance between the tubes grows, the amplitude of the variable component of the vortex sheet intensity also grows, while the magnitude of the constant component drops (see the lower part of Fig. 3). It hence follows that as the grid compactness increases the magnitude of the exciting force caused by vortex separation diminishes. This fact is also confirmed experimentally [8]. However, it is necessary to note that another excitation mechanisel associated with the mutual tube displacement also appears in compact grids.


Fig. 2. Dependence of the dimensionless frequency of vortex separation on the relative compactness of the tube grid: 1) experiment for a corridor tube bundle with a longitudinal spacing $Z / r=6-\infty$ according to data in [8]; 2) experiment [8] for checkerboard tube bundles; 3) computation for a tube grid.
Fig. 3. Change in intensity of the trailing vortex sheet on the dimensionless time.

## NOTATION

Re $=u_{\infty} d / v$, Reynolds number; $u_{\infty}$, free stream velocity; d, tube diameter; $v$, fluid kinematic viscosity; t, tube grid step; $\beta$, phase shift during vortex collapse from tube to tube; $\alpha$, angle of vortex layer separation from a tube; $u$ and $v$, stream velocities in the $x$ and $y$ axes directions, respectively; $\Delta \Gamma$, intensity of the i-th vortex in the stream; $N$, number of vortices on a tube; $S h=f d / u$, dimensionless frequency of vortex separation; f, frequency of vortex separation; $\tau=\tau u_{\infty} / r$, dimensionless time; and $\eta$, distance between tubes of the bundle in the stream direction.

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Choice of parameters of high-Temperature jet recovery units
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The results of mathematical modeling of the process of jet heat transfer are presented. Nomograms are constructed for the technical-economic analysis of the design.

The sharp intensification of heat transfer accompanying the flow of a jet of liquid (or gas) onto an interface is being increasingly used in the design of diverse heat-engineering systems and units. A number of designs of jet recovery units [1] have been proposed in recent years. Among these units the modular jet recovery units designed at the Gas Institute of the Ukrainian SSR Academy of Sciences are most widely used [2].

The most important feature of systems with jet blowing is that their basic parameters vary over a wide range. This makes it impossible to compare existing experimental data on convective heat transfer at jet impact [3-5] and limits their applications in the design of jet systems.

In this work results were obtained based on the numerical solution of the system of differential equations of conservation of momentum, mass, and energy, closed with the help of the two-parameter $k-\varepsilon$ model of turbulence.

The working scheme of the model is shown in Fig. 1. We are studying a separate opening (nozzle) with a diameter of $d_{o p}$ in a perforated plate and a cylindrical region of radius $R$ surrounding it into which an air jet with an effective cross section $d_{0}$ and velocity $U_{0}$ (the rate of flow of air through the nozzle is given by $\rho_{0} U_{0} \pi d_{0}^{2} / 4$ ) flows. Since in this case for a correctly designed jet system the effect of the drifting flow on the hydrodynamics and heat transfer should be small, it is ignored in the formulation of the problem. The heat-transfer surface, which the jet strikes, is defined in terms of the equivalent radius R from the condition $\pi R^{2}=F / N$ ( $F$ is the total area of the heat-transfer surface and $N$ is the total number of openings in the perforated plate). The boundary conditions for the temperature were set so as to take into account the effect of the recirculating air on the character of the heat transfer. The temperature of the recirculating flux at the inlet into the working region (top part of the boundary 2 in Fig. 1) was assumed to equal the mean temperature $T_{t b}$ of the heated air leaving the volume (bottom part of boundary 2). This value of the temperature was also used for the surface of the perforated plate, and a constant temperature $T_{p}$ was given on the heat-transfer surface 4. On the remaining boundaries (axis of symmetry and the bottom part of the boundary 2 through which the air leaves the working region) the condition $\partial \mathrm{T} / \partial \mathrm{r}=$ 0 was imposed.

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